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13 Your Roll No. 2023

Sr. No. of Question Paper : 5006

Unique Paper Code : 62354443

Name of the Paper : Analysis (LOCF)

Name of the Course : **B.A. (Prog.)**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory.

3. Attempt any two parts from each question.

4. **All** questions carry equal marks.

1. (a) Define limit point of a set $S \subseteq \mathbb{R}$. Find the limit points of the following sets :

P.T.O.

(i) $\mathbb{N}$

(ii) $\mathbb{R}$

(b) Define closed set. Prove that the union of finite number of closed sets is closed set.

(c) If A and B are non-empty bounded above subsets of $\mathbb{R}$ ans $C = \{x + y | x \in A, y \in B]$ then show that : $Sup(C) = Sup(A) + Sup(B)$.

(d) Define neighborhood of a point and an open set. Give an example of each of the following :

(i) A non-empty set which is a neighborhood of each of its points with the exception of one point.

(ii) A non-empty set which is neither an open set nor a closed set.

(iii) A non-empty closed set which is not an interval.

(iv) A non-empty open set which is not an interval.

2. (a) Test the continuity of function

$$f(x)=\begin{cases}\dfrac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}, & x\neq 0\\ 0, & x=0\end{cases},$$ at $x = 0$.

(b) Show that the function f defined by $f(x) = x^2$ is uniformly continuous on $[-2,2]$.

(c) Prove that the union of an arbitrary family of open sets is an open set.

(d) Show that every continuous function on a closed interval is bounded.

3. (a) Show that a sequence cannot converge to more than one limit.

(b) Show that the sequence $<a_n>$ defined by :

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots\cdots + \frac{1}{n+n}, \ \forall \ n \text{ converges.}$$

(c) State Cauchy convergence criterion for sequences and hence show that the sequence $<x_n>$ defined by :

$$x_n = 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots\cdots + \frac{1}{2n-1},$$

does not converge.

(d) Show that every convergent sequence is bounded but the converse is not true.

4. (a) State Leibnitz test for convergence of an alternating series : $\sum_1^{\infty}(-1)^{n-1}u_n \;\forall\, n$ and test the convergence and absolute convergence of the series :

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}\cdots\cdots\cdots .$$

(b) Check the convergence of the following series:

$$\sum_{n=1}^{\infty}\frac{3.6.9.\cdots 3n}{7.10.13.\cdots(3n+4)}x^n \ (x>0).$$

(c) Show that the sequence $<x_n>$ defined by :

$x_1 = 1,\ x_{n+1} = \dfrac{3+2x_n}{2+x_n},\ n \geq 2$ is convergent. Also find $\lim_{n\to\infty} x_n$.

(d) Test the convergence of the series whose n^{th} term is $\left(\sqrt{n+1}-\sqrt{n}\right)$.

5. (a) If $<a_n>$ and $<b_n>$ are sequences of real numbers such that

$\lim_{n\to\infty} a_n = a$, $\lim_{n\to\infty} b_n = b$ then prove that :

$$\lim_{n\to\infty} (a_n b_n) = ab.$$

(b) Define Riemann integrability of a function. Show that x^2 is integrable on any interval $[0, k]$.

(c) Show that $\lim_{n\to\infty} \frac{1}{n}\left[1+2^{\frac{1}{2}}+3^{\frac{1}{3}}+\cdots\cdots+n^{\frac{1}{n}}\right]=1$.

(d) Define the sum of a convergent series. Find the sum of the following series :

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}+\cdots\cdots$$

6. (a) Test the convergence and absolute convergence of the series :

(i) $\sum \frac{(-1)^{n-1}}{n^2}$.

(ii) $\sum \frac{(-1)^{n-1}}{n\sqrt{n}}$.

(b) Let $<a_n>$ be a sequence defined by

$$a_1 = 1,\ a_{n+1} = \frac{(2a_n + 3)}{4},\quad \forall\, n \geq 1,$$

Prove that $<a_n>$ is bounded above and monotonically increasing. Also find $\lim_{n\to\infty} a_n$.

(c) Prove that every continuous function is integrable.

(d) Discuss the convergence of the series :

$$\sum_{n=1}^{\infty} \frac{\sin nx + \cos nx}{n^{3/2}} .$$

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(14) Your Roll No. 2023

Sr. No. of Question Paper : 5025 E

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Name of the Course : **B.A. (Prog.)**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

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2. **All** questions are compulsory.

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1. (a) Let S be a non empty bounded set in $\mathbb{R}$. Let $a > 0$, and let $aS = \{as : s \in S\}$. Prove that $\inf aS = a \inf S$, $\sup aS = a \sup S$.

P.T.O.

(b) Define order completeness property of real numbers.

(c) Define limit point of a set. Show that the set N of natural numbers has no limit point.

(d) State and prove Archimedean property of real numbers.

2. (a) Show that the function defined as

$$f(x)=\begin{cases} x, & \text{if } x \text{ is irrational} \\ -x, & \text{if } x \text{ is rational} \end{cases}$$

is continuous only at $x = 0$.

(b) Show that the function f defined by $f(x) = x^2$ is uniformly continuous on $[-2,2]$.

(c) Define an open set. Prove that every open interval is an open set. Which of the following sets are open.

(i) $]2, \infty[$

(ii) $[3,4[$

(d) Let A and B be bounded nonempty subsets of R, and let $A + B = \{a + b: a \in A, b \in B\}$. Prove that $\sup (A + B) = \sup A + \sup B$.

3. (a) Prove that every convergent sequence is bounded. Justify by an example that the converse is not true.

(b) Prove that the sequence $\langle a_n \rangle$ defined by the recursion formula :

$$a_1 = \sqrt{7}, a_{n+1} = \sqrt{7 + a_n}$$

converges to the positive root of $x^2 - x - 7 = 0$.

(c) State Cauchy's convergence criterion for sequences. Check whether the sequence $\langle a_n \rangle$, where

$$a_n = 1 + \frac{1}{5} + \frac{1}{9} + \cdots \cdots\cdots \ldots + \frac{1}{4n-3}$$

is convergent or not.

(d) Test for convergence the series :

$$1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \cdots\cdots\cdots\cdots\cdots\cdots$$

4. (a) Prove that, if the series $\sum u_n$ converges, then $\lim_{n\to\infty} u_n = 0$. Show by an example that the converse is not true.

(b) Test for convergence the series :

$$\sum_{n=1}^{\infty} \frac{2.4.6 \ldots\ldots\ldots (2n+2)}{3.5.7 \ldots\ldots\ldots (2n+3)} x^{n-1} \qquad (x > 0)$$

(c) Let $\langle a_n \rangle$ be a sequence defined by :

$$a_1 = 1, \; a_{n+1} = \frac{3 + 2a_n}{2 + a_n}, \quad n \geq 1.$$

Show that $\langle a_n \rangle$ is convergent and find its limit.

(d) Prove that a sequence of real numbers converges if and only if it is a Cauchy sequence.

5. (a) State Leibnitz test for convergence of an alternating series of real numbers. Apply it to test for convergence the series

$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\dots\dots\dots\dots\dots\dots\dots\dots$$

(b) Show the sequence defined by $\langle a_n \rangle = \langle n^2 \rangle$ is not a Cauchy sequence.

(c) Prove that the sequence $\langle a_n \rangle$ defined by the relation,

$$a_n = 1,\ a_n = 1+\frac{1}{1!}+\frac{1}{2!}+\dots\dots\dots\dots+\frac{1}{(n-1)!},\quad (n \geq 2),$$

converges.

(d) Prove that every continuous function is integrable.

6. (a) Define Riemann integrability of a bounded function f on a bounded closed interval [a, b]. Show that the function f defined on [a, b] as

$$f(x)=\begin{cases}0 & \text{when x is rational}\\ 1 & \text{when x is irrational}\end{cases}$$

is not Riemann integrable.

(b) Test for convergence the series

$$\sum_{n=1}^{\infty}(-1)^n \frac{\cos n\alpha}{\sqrt{n^3}}, \ \alpha \text{ being real.}$$

(c) State D'Alembert's ratio test for the convergence of a positive term series. Use it to test for convergence the series $\sum_{n=1}^{\infty}\frac{n!}{n^n}$.

(d) Show that $f(x) = \frac{1}{x}$ is not uniformly continuous on $[0,1]$.